Eliminating the Effects of the Company Insolvency Risk: A Model Approach

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Abstract

Managers around the world identify risk of liquidity problems as the primary barrier of their decisions. In the global economy, in the turbulent environment, the effects of companies' insolvency risk lead to many operational and strategic problems also causes of contagion effect. Therefore, it is important to study the phenomenon of insolvency, determinants that shape it as well as seek opportunities to reduce the risk of this problem. The aim of the study is to identify the determinants that affect the risk of insolvency of companies in Poland as well as propose a model of the effects of companies' insolvency based on the profiles of risk factors and the actions as well as instruments to successfully manage the risks of default. We used Logit Model and Panel Data. Our findings suggest that appropriate debt management, as well as the need for synchronization of receipts and expenditure, which should secure the current payments in a short run is crucial in identifying and reducing the risk of insolvency, in a short term (in relation to all groups of companies). However, in the longer term, consideration should be given to the different factors that can allow the identification of the risk of insolvency, which may further allow the entrepreneurs to make quick response, or spread out over time the operations (e.g. restructuring) and thus allow avoiding the insolvency. Hence, in private companies a sustainable performance should be instituted by the board of directors.

Keywords: Insolvency, Risk, Company

JEL Codes: G, G3, G32

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1. **Introduction**

Nowadays assessment of economic situation of Small and Medium Enterprises (SME) has become an inherent part of business environment which has experienced dynamic changes in economic activity. One of the most important step is to identify early signals of financial distress or insolvency. This can prevent bankruptcy.

The deterioration process of the financial condition should be understood as a process of deterioration with varying intensity of key financial indicators that determines the position of a company in the various stages of this process. It contains financial uncertainty, financial disorder or bankruptcy (Porada-Rochoń, 2013). However it should be noted that among these stages the limits are of the contractual nature and the occurrence of the insolvency may take place both in the stage of financial disorder and bankruptcy.

Zhuang and Chen (2014) pointed out that the financial state of a company often cannot be observed directly, but only some signal indicators associated with the financial state can be observed. Therefore managers should implement complex processes (different methods and instruments) to prevent the insolvency risk.

2. **Insolvency risk and the ways of minimizing It – Theoretical Approach**

According to Oxford English Dictionary, Insolvency means the fact of being unable to pay one’s debts or charge one’s liabilities. Hotchkiss (2008) defines the situation of a firm being in difficulty as "the inability of the company to pay its debts or financial imbalance between funding sources and economic means to be covered in them" depending on the size of the financial risk which it assumes into the economic activity.

Lupu (2014) summarizes that firm in difficulty is “the company facing a lack of liquidity and difficulties arising from the fulfillment of obligations outstanding financial previously contracted necessary for smooth running of its activity”.

It is quite common to use some related terms instead of insolvency: financial difficulties, financial distress, even failure or bankruptcy, although the terminological meaning is different.

Altman (1971) was among the first to make the distinction between the terms "failure", "insolvency" or "bankruptcy". The term failure means the inability to achieve an adequate return on investment. The company can be operational for years before they cease their business operations. Insolvency means that the company cannot pay its liabilities when they fall due, which may be a temporary situation (called technical insolvency), while permanent insolvency means that the liabilities exceed the value of company assets. Bankruptcy has a legal meaning. As White 1989 mentioned, insolvency doesn’t mean economic viability and should be considered releasable.

The risk is related to the business activities. The risk management takes a risk as a key element in terms of its identification, evaluation, minimizing the potential effects and elimination. The issue of the risk of insolvency and bankruptcy is widely presented in the literature. By analyzing the concept of risk it can be indicated that the risk in a general sense, means "the variation of the result obtained under the environmental pressure, it represents the potential damage incurred on: assets, heritage, interests and economic activities" (Mihai, 1997). Corporate insolvency risk is defined as the probability that a company will become insolvent in the next 12 months (Rohan et al. 2007).
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<th>Endogenic</th>
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<td>• Inadequate budget development</td>
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<td>• Capital inadequacy</td>
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<td>• Lack of adequate capital reserves</td>
<td>• The characteristics of the environment: economic, financial, fiscal,</td>
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<td>• Limited budget monitoring</td>
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<td>• Too high level of debt</td>
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<td>• High costs</td>
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<td>• Increased liquidity</td>
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<td>• Repeated losses in the operating activity</td>
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<td>• Lack of monitoring of cash</td>
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<td>• Less competitive products and services</td>
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<td>• Inadequate and ineffective communication</td>
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<td>• Uncertainty about future project returns</td>
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<td>• The lack of an early warning system</td>
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Source: own elaboration on literature review (Argenti, 1976; Aguiar-Díaz, Ruiz-Mallorquí, 2015; Porada-Rochon, 2015)
Many studies on the phenomenon of the insolvency risk including the risk of bankruptcy (often indicated as identical) indicate the presence of endogenic and exogenic risk factors. These factors can be summarized as given Table 1.

To resolve problems of insolvency and eliminate the effects of the Companies' Insolvency Risk, two group of actions should be taken into consideration:

a) Prognostic/ Diagnostic;
b) Intervention.

The first group of actions involve method of score functions used to provide models for assessment/predicting the insolvency risk as well as financial distress and bankruptcy (Beaver 1966, Altman 1971, Beerman 1976, Sulphey and Nisa, 2013). Models are based on discriminatory analysis. One of the first step is a selection of appropriate financial ratios for assessment financial health.

The second group of actions, Intervention, depends on the causes and symptoms of insolvency risk. One of the most important step to be taken is turnaround process. Usdin and Bloom (2012) recommended the use of professionals who are independent of the pressures inherent in managing a company (including internal, emotional or family pressures) and are available to analyse and advise a company in a difficult financial position. Such professionals include Turnaround specialists, Attorneys specializing in advising distressed companies, and Accountants specializing in assisting distressed companies. Apart from that, according to the research done by Porada Rochon, 2013, there are many turnaround processes (Schendel et al., 1976, Hambrick, Schecter 1983, Zimmerman 1991, Sudarsanam, Lai 2001 Smith, Graves 2005), which are characterized by a high degree of the diversification in terms of content, form and the degree of fine detail. In addition, there is a clear differentiation in terms of the role of tools and activities within the framework of the financial decisions. It is therefore extremely difficult to build a turnaround process aimed at eliminating the effects of the risk of insolvency. However, proper identification of the causes and sources of dysfunction makes it easy to select the appropriate configuration tools and actions of a stabilizing or prospective approach.

Elimination of the effects of insolvency risk depends on the specific dysfunctions of the financial conditions of the object under consideration. The most important question is how a company is able to face risk factors that appear on the market and how a company is able to cope with risk.

Running a business requires to have assets that determine the choice of a variety of financing sources. In accordance with the prudential norm of financing, long-term or fixed assets should be financed with long-term sources like equity capital, preference capital, debentures, term-loans, etc., and current assets are to be financed with short-term sources. In other words, fixed and other long-term assets are typically not financed with short-term sources. Asset composition analysis is a significant measure of evaluating the risk exposure of a company’s capital structure (Bardia 2012). Elliehausen and Wolken (1993) argue that companies with a high short-term debt ratio will have less access to institutional finance because a higher short-term debt ratio is generally associated with the presence of higher financial risk.

Shleifer and Vishny (1992) discuss the nature of asset illiquidity especially in the context of distress firms. They argue that when firms have trouble meeting debt payments and sell assets, the highest valuation potential buyers of these assets are likely to be other firms in the same industry. But with the possibility of a contagion effect, these firms themselves are likely to have trouble with debt payments. The other probable group of buyers of these assets,
industry outsiders, would face agency costs of hiring specialists to run these assets and may fear overpaying for lack of proper knowledge of the assets characteristics. Hence when industry buyers cannot buy the assets and industry outsiders face significant costs of acquiring and managing the assets, assets in liquidation fetch prices below value in best use, which is the value when managed by specialist (Flagg et al. 2011).

Guney et al. (2003), Ferreira and Vilela (2004) and Ozkan and Ozkan (2004) argued that firms in financial distress could raise their cash levels in order to reduce their default risk. However, Kim et al. (1998) found that firms with financial distress tend to have lower levels of liquidity, mostly due to the fact, that firms with problem with payment cannot accumulate cash (Garcia-Teruel, Martinez-Solano, 2008).

Another important issue is the asymmetric information which leads to a higher cost for external sources of funds and credit rationing for companies, because of conflict of interests between stakeholders (Myers, 1977). This conflict can lead to a problem of underinvestment, given the priority of creditors in case of bankruptcy. Moreover, shareholders also have incentives to issue new debt, which increases risk and lowers the value of existing debt. As a consequence, creditors demand a higher risk premium. Asymmetric information is particularly evident between internal stakeholders in the firm and external stakeholders-potential investors. Therefore, it results in a higher cost for external sources of funds. Because of that firms give priority to resources generated internally over debt and new equity, according to the pecking order theory (Myers, 1984). Wallace et al (1994) took into consideration the correlation between capital scale and information transparency. They found that between those above is positive relationship, that means, the more transparent the information is, and the lower the capital cost is. Li (2005) study testified that the company in financial distress might have diminished information disclosure previously to lower its transparency (Shyan–Rong Chou et. al, 2010)

Apart from above actions, there also exists a legal procedure. Several Member States are currently reforming their insolvency laws with a view to improving the legal framework enabling the early restructuring of companies in financial difficulty. There is a risk that a lack of coordination of these reforms as well as a lack of action on the part of those Member States, which do not have effective frameworks in place or plans to reform their laws, will be a missed opportunity for removing barriers to the internal market which flow from the divergence of insolvency laws (European Commission, 2014)

3. Purpose and methodology of the study

The purpose of the study is to determine the risk factors of insolvency for groups of enterprises with varying levels of financial security in Poland. We choose Poland because Poland is one of the countries at risk of losing financial liquidity. This research takes into consideration problems that plagues Polish entrepreneurs which are: payment defaults and significant delays in receiving receivables. It increases the money gap that forces entrepreneurs to collect cash from other sources (eg. Loans) or maintain higher reserves of own funds. This in turn generates additional costs, as well as immobilization of cash. At the same time some entrepreneurs cannot afford to introduce such solutions. Therefore, problems with liquidity are transformed to insolvency and often led as a consequence to corporate bankruptcy. Next problem that occurs is domino effect: liquidity problem of one company influence on the other companies.

There are limitations in the lack of direct reference of the results of our research to the research of authors from other countries, Central and Eastern Europe, the Old Continent,
The United States etc. Some researchers are conducted in different countries for various research trials. Therefore, they are specific and cannot be generalized.

It was found that the companies that satisfy the condition of financing fixed assets with equity have a very secure financial situation— in the long term— and therefore are not affected by the increase in the risk of insolvency even in a situation of temporary liquidity problems. In turn, those entities that do not have the equity to finance the assets with slowest term of converting into cash are much more exposed to the risk of the insolvency. Therefore it is possible to establish the determinants that increase the risk of insolvency and further to prevent risks of financial stability and/or to eliminate them. This research was conducted for two separate groups of companies: small and medium-sized entities. Therefore, the following models for the subsequent groups have been developed:

a) small-sized enterprises not maintaining the golden rule of accounting (E/FA<1) - Model 1a.  
b) small-sized enterprises maintaining the golden rule of accounting (E/FA>=1) - Model 1b.  
c) medium-sized enterprises not maintaining the golden rule of accounting (E/FA<1) - Model 2a.  
d) medium-sized enterprises maintaining the golden rule of accounting (E/FA>=1) - Model 2b.

where E is Equity and FX is Fixed Assets.

On identifying the risk factors for the insolvency as a dependent variable the relationship of equity to assets has been adopted. In this study, the following research hypothesis were formulated: the different factors determine to provide the golden rule of accounting in enterprises depending on whether they are at higher risk of insolvency or not. Due to the two samples of enterprises (SME) The hypotheses are:

\( H_M \) - Different factors determine the behaviour of the golden rule of the financing of small-sized enterprises depending on whether they are at higher risk of insolvency or not.  
\( H_S \) - Different factors determine the behaviour of the golden rule of the financing of medium-sized enterprises depending on whether they are at higher risk of insolvency or not.

Collected data consist of two type of data: survey data and financial and economic panel data. There were analysed separately. Survey database contains 100 observations. The micro-panel database including aggregate information on an annual basis for the period from 2007 to 2011 for 265 small-sized companies and 176 medium-sized enterprises contains 2205 observations. To analyse survey data logistic regression has been used and to analyse the panel data set the linear regression for panel data with random effect model has been used. For this empirical study we used dependent variables: Equity and Fixed Assets; as well as logarithm of independent variables: Cash flow, Creditors, Other Current Assets, Shareholders funds th EUR, Receivables To Assets Total th EUR, Other Shareholders Funds, PL before tax, PL after tax, Ratio Turnover Assets, Debtors, Current Assets to Total Assets, Ratio Turnover Assets.

This research was carried out in R.

The logistic regression or the logit regression is one of the many methods of describing the relationship between a set of independent variables and dependent variable. The basic use of the logistic regression is to apply this method to a binary (i.e. zero-one) dependent variable (Larose, 2008).
The logit model is a special case of the generalized linear model. The relationship between the explanatory variables and probability is as follows:

\[ \ln \frac{p(1|x)}{1-p(1|x)} = \alpha + \beta^T x, \] (1)

where \( p(1|x) \) is the probability of the value 1 of the dependent variable, \( \alpha \) is constant in the model, \( \beta \) is a vector of parameters, \( x \) is a matrix of explanatory variables and \( \ln \frac{p(1|x)}{1-p(1|x)} \) is called logit. Formula (1) can be converted to form as:

\[ p(1|x) = \frac{\exp(\alpha + \beta^T x)}{1 + \exp(\alpha + \beta^T x)} \] (2)

and

\[ p(0|x) = \frac{1}{1 + \exp(\alpha + \beta^T x)}. \] (3)

The function presented on the left side of the equation (1) denoting by \( \text{logit}(v) \) and is referred to as logit function, whereas the model given by equation (2) is referred to as logistic regression or the logistic model (Koronacki and Cwik, 2008). The model is a linear model with respect to the estimated parameters \( \beta \) and explanatory variables \( x \). Logit is the logarithm of the odds. In the case of equal odds the value of logit is equal to zero (Gruszczynski et al., 2009).

The estimator of parameters \( \beta \) in the logit model is the method of maximum likelihood. This means maximizing the likelihood function of the following form:

\[ l(\beta|x) = \prod_{i=1}^{n} \hat{p}(1|x_i)^{y_i} \hat{p}(0|x_i)^{1-y_i}, \] (4)

where \( y_i \) is binary dependent variable of \( i \)-th observation. Maximization of the function given with formula (4) is iterative with respect to the parameters \( \alpha \) and \( \beta \) (Koronacki and Cwik, 2008).

A typical coefficient of determination \( R^2 \) applied in the logic regression models is called McFadden pseudo – \( R^2 \) coefficient. It is given by the following formula:

\[ \text{pseudo} - R^2 = 1 - \frac{\ln L_{MP}}{\ln L_{MZ}}, \] (5)

where \( \ln L_{MP} \) is a logarithm of likelihood function for a full model, and \( \ln L_{MZ} \) is a logarithm of likelihood function for a model reduced to the constant. The pseudo – \( R^2 \) is used generally for comparison of not nested logit models for the same variable (Gruszczyński et al., 2009).

The coefficient of determination which demonstrate the prognostic quality of the model is the counting \( R^2 \). The value of such forecasts is generally presented by using the tables of accuracy. This coefficient is calculated as the share of cases with accurate forecasts in the total number of observations (Gruszczyński et al., 2009).

The test of the significance of each parameter in the model is a so called Wald test. The test statistic is given by the formula:

\[ Z_{Wald} = \frac{\hat{\beta}_i}{SE(\hat{\beta}_0)}, \] (6)
where \( \hat{\beta}_i \) is the estimator of the parameter, and \( \text{SE}(\hat{\beta}_i) \) denotes the standard error of \( \hat{\beta}_i \). Under the null hypothesis \( (\hat{\beta}_i = 0) \) the statistic has the standard normal distribution (Koronacki and Cwik, 2008).

The panel data set consists of observations on individuals and each individual is observed the same number of times (e.g. subsequent periods). Each unit in the data set is observed in specific time periods e.g. months, years, etc. The size of the panel concerns two dimensions: the number of units surveyed in a data set (N) and the number of periods in each of these units tested (T). Therefore, the variables in panel data have a double notation such that for the implementation of the dependent variable in the model it is stated as \( y_{it} \) \( (i = 1, N; t = 1, T) \) (Gruszczynski et al., 2009). Hence the panel data structure is much effective than one-dimensional data. The two-dimensional nature of the data allows for the analysis of both sections at the same time (Kopczewska et al., 2009).

The main advantage of the use of panel data in modelling is the ability to remove the burden of the estimator that arises from the error of omitted variables. Taking into consideration the Cobb–Douglas production function as given in the following form:

\[
y_i = \mu + x_i^T \beta + \varepsilon_i,
\]

where \( y_i \) is the logarithm of the production volume, \( x_i \) is the vector of logarithmic transformed effort of manufacturing factors, \( \varepsilon_i \) is a random component in the model, and \( \mu \) denotes a constant in the model.

Mundlak (1961) showed that using the same input factors, enterprises can obtain different levels of profit. The main reasons are different skills of managing. The cross-sectional nature of the data makes it impossible to identify this effect. The number of effects is equal to the number of units so it becomes a part of the random component. The effect of the manager skills is also correlated with \( x_i \). The high level of skills of a manager usually results in high profits or high values of the variable \( y_i \) and these will lead to increased investment in \( x_i \). Therefore a random component \( \varepsilon_i \) and variables \( x_i \), become correlated which in the case of cross-sectional data leads to inconsistency of the estimator. This effect is called a Mundlak effect. The use of panel data allows to enter the effect of the skills of manager in the model. The model then takes the following form:

\[
y_{it} = m_i + x_{it}^T \beta + \varepsilon_{it},
\]

where \( m_i \) is the manager skills effect. It is assumed that it is a constant in time, has an impact on the \( y_{it} \) and is related to \( x_{it} \). Introducing the manger effect into model eliminates bias of the estimator (Gruszczynski et al., 2009).

The one–way linear model for panel data has the following form:

\[
y_{it} = \alpha_i + x_{it}^T \beta + \varepsilon_{it},
\]

where \( y_{it} \) denotes the dependent variable, \( x_{it} \) is a vector of explanatory variables, \( \alpha_i \) is the \( i \) – th individual effect, \( \beta \) is a vector of the structural parameters and \( \varepsilon_{it} \) is a random component of model (Gruszczynski et al., 2009).

The individual effect in the model contains all the information about each unit that are fixed at the time and have an impact on the depended variable \( y_{it} \), however they are not included in the explanatory variables \( x_{it} \), as this is difficult or impossible to measure or describe them. The individual effect also includes the manager effect described above (Gruszczynski et al., 2009). Depending on the treatment and the estimation of the individual
The one–way model along with random effects is given by the formula:

\[ y_{it} = \mu + \alpha_i + x_{it}^T \beta + \varepsilon_{it}, \]  

(8)

where \( y_{it} \) indicates dependent variable, \( x_{it} \) indicates explanatory variables vector, \( \alpha_i \) indicates the individual effect for each considered unit, \( \beta \) is the vector of structural parameters, \( \varepsilon_{it} \) is a random component, and \( \mu \) is the constant (Verbeek, 2004). The presence of the constant term in the equation (8) distinguishes it from typical model with fixed effects and arises from the nature of the individual effects in the model with random effects. The effects are treated as random, therefore do not constitute additional parameters to estimate as it happens in the case of models with fixed effects. Hence the model form (8) is to be treated as:

\[ y_{it} = \mu + x_{it}^T \beta + v_{it}, \]

\[ v_{it} = \alpha_i + \varepsilon_{it}, \]

that \( i = 1, N, t = 1, T \), where the element \( v_{it} \) provides the sum of the random individual effects (i.e. \( \alpha_i \)) and white noise (i.e. \( \varepsilon_{it} \)). As a result, in contrast to the linear models with fixed effects, in the models with random effects, there is no problem of colinearity among constant in time independent variables and binary variables representing manager effects (Gruszczynski et al., 2009).

In the random individual effect approach, \( \varepsilon_{it} \sim \text{IID}(0, \sigma^2 \varepsilon) \) and strict exogeneity of the explanatory variables i.e. \( E(x_{it} \varepsilon_{is}) = 0 \), where each \( i = 1, N \) and \( t, s = 1, T \) are assumed. The placement of the individual effects in the stochastic part of the model forces additional assumptions (Gruszczynski et al., 2009):

- for each unit the distribution of individual effects is \( \alpha_i \sim \text{IID}(0; \sigma^2 \alpha) \)
- the independence of individual effects \( \alpha_i \) from independent variables \( x_{it} \) for any \( i, j = 1, N \) and \( t = 1, T \) in order to avoid the problem with endogenous nature
- the independence of individual effects \( \alpha_i \) from the random component of the model \( \varepsilon_{it} \) for all units \( i \) and in all periods \( t \), i.e. \( E(\varepsilon_{it} \alpha_j) = 0 \), for any \( i, j = 1, N \) and \( t = 1, T \).

Taking into account the above assumptions provides that the OLS estimator used to estimate the model (8) is consistent and unbiased but is not effective. The lack of this estimator efficiency results from the decomposition of \( v_{it} \), which is the sum of the random individual effects (i.e. \( \alpha_i \)) and white noise (i.e. \( \varepsilon_{it} \)) (Gruszczynski et al., 2009).

The variance of the individual effects has a non-zero value, so an OLS estimator will not be the most effective in-class estimators. The solution tough is to use an estimator of generalized least squares (GLS) method. Assuming that \( v_i \) is a sum of the individual effects of random vector (\( \alpha_i \)) and white noise (\( \varepsilon_{it} \)) for each unit:

\[ v_i = \begin{bmatrix} \alpha_i + \varepsilon_{i1} \\ \vdots \\ \alpha_i + \varepsilon_{iT} \end{bmatrix} = \alpha_i + \varepsilon_i, \]  

(9)
\( t \) is a \( T \times 1 \) vector of ones, \( \varepsilon_i \) is a \( T \times 1 \) vector of error terms, \( I_T \) is an identity matrix with dimension \( T \times T \) and \( u^T \) is a \( T \times T \) dimension matrix with all elements equal to 1, it provides to the following equation:

\[
E(v_i v_j) = E(\alpha_i t + \varepsilon_i) (\alpha_j t + \varepsilon_j)^T = 0 \quad \text{for} \quad i \neq j,
\]

and

\[
E(v_i v_j) = E(\alpha_i t + \varepsilon_i) (\alpha_j t + \varepsilon_j)^T = \sigma^2_\alpha u^T + \sigma^2_\varepsilon I_T = \omega \quad \text{for} \quad i = j,
\]

where \( \omega \) is a \( T \times T \) matrix. The variance – covariance matrix \( \Omega \) of the random component which is the sum of the random individual effects (i.e. \( \alpha_i \)) and white noise (i.e. \( \varepsilon_{it} \)) necessary in the construction GLS estimator is the \( NT \times NT \) dimension matrix:

\[
\Omega = \begin{bmatrix}
\omega & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \omega
\end{bmatrix}
\]

The inverse matrix \( \Omega^{-1} \) of the matrix \( \Omega \) given by the following formula:

\[
\Omega^{-1} = \begin{bmatrix}
\omega^{-1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \omega^{-1}
\end{bmatrix}
\]

and has \( NT \times NT \) dimension. The inverse matrix of the variance - covariance matrix \( \Omega^{-1} \) of \( NT \times NT \) dimension, may be presented in the form:

\[
\Omega^{-1} = \frac{1}{\sigma^2_\varepsilon} I_T + \frac{1}{T} u^{-1}(\psi - 1)
\]

where

\[
\psi = \frac{\sigma^2_\alpha}{\sigma^2_\varepsilon + T \sigma^2_\alpha}
\]

(Gruszczyński et al., 2009). Regarding \( \bar{y} \) as the average of dependent variable for all units in all considered periods and the \( \bar{x} \) vector as the average of explanatory variables for all units in all considered periods, substituting (14) to the formula for the GLS estimator, the following formula for random effects estimator is provided (Gruszczyński et al., 2009):

\[
\hat{\beta}_{RE} = \left( \sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^T + \psi T \sum_{i=1}^{N} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^T \right)^{-1}
\]

\[
\left( \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - \bar{y}_i)(y_{it} - \bar{y}_i)^T + \psi T \sum_{i=1}^{N} (\bar{x}_i - \bar{x})(\bar{y}_i - \bar{y})^T \right)
\]

where \( \hat{\beta}_{RE} \) is a vector of parameters and the variance - covariance matrix given by the formula (Gruszczyński et al., 2009):

\[
V(\hat{\beta}_{RE}) = \sigma^2_\varepsilon \left( \sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{it} - \bar{x}_i)^T + \psi T \sum_{i=1}^{N} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^T \right)^{-1}
\]

The value of \( \sigma^2_\varepsilon \) and \( \sigma^2_\alpha \) is unknown as a consequence it is impossible to designate the exact value of the estimations obtained by using the GLS estimator. Specifying the estimate of \( \sigma^2_\varepsilon \) and \( \sigma^2_\alpha \) along with substituting it into the formula (16) an estimate of the Feasible GLS is obtained, which is an estimator of random effects. The estimation of \( \sigma^2_\alpha \) requires a prior estimate of an additional model using the inter-group estimator (Gruszczyński et al., 2009).

The main assumption in random effects approach is the independence of explanatory variables and individual effects. The individual effects become the part of the random
component and their relationship with independent variables leads to a loss of consistency by the estimator. (Gruszczynski et al., 2009).

4. Findings

Table 1 shows the results of the model analysis for small-sized enterprises in two versions: Model 1a. for those entities that do not have maintained the golden rule of financing and Model 1b. for the group of companies that meet the golden rule.

Table 1: The results of model analysis for small-sized enterprises.

**Model 1a.** The small-sized enterprises that meet the golden rule of accounting (E/FA<1)

| Coefficients                          | Estimate | Std. Error | t value | Pr(>|t|) | Significance |
|---------------------------------------|----------|------------|---------|----------|--------------|
| (Intercept)                           | -0.090   | 0.125      | -0.720  | 0.472    |              |
| Cash flow                             | 0.002    | 0.001      | 1.652   | 0.099    |              |
| Cash Flow (Lag: -1)                   | -0.004   | 0.001      | -2.780  | 0.006**  |              |
| Creditors                             | -0.0004  | 0.000      | -2.300  | 0.022*   |              |
| Other Current Assets                  | 0.001    | 0.000      | 2.224   | 0.026*   |              |
| Shareholders’ Funds (Lag: – 1)        | 0.001    | 0.000      | 3.214   | 0.001**  |              |
| Receivables to Assets (Lag: – 1)     | 0.010    | 0.001      | 5.262   | 0.000*** |              |
| Other Shareholders Funds (Lag: -1)    | -0.001   | 0.000      | -3.992  | 0.000*** |              |
| Profit / Loss before taxation         | -0.002   | 0.001      | -1.804  | 0.072    |              |
| Profit / Loss before taxation (Lag: -1) | 0.004   | 0.000      | 3.071   | 0.002**  |              |
| Turnover Assets Ratio                 | 0.001    | 0.000      | 4.159   | 0.000*** |              |
| Debtors                               | -0.0005  | 0.000      | -1.819  | 0.069    |              |

Total Sum of Squares: 448.07
R-Squared : 0.094471
F-statistic: 7.54951 on 11 and 796 DF
theta: 0.4118

Residual Sum of Squares: 405.74
Adj. R-Squared : 0.093068
p-value: 1.898e-12
All variables are in logarithm.

**Model 1b.** The small-sized enterprises which meet the golden rule of accounting (E/FA>=1)

| Coefficients                          | Estimate | Std. Error | t value | Pr(>|t|) | Significance |
|---------------------------------------|----------|------------|---------|----------|--------------|
| (Intercept)                           | -1.493   | 2.720      | -0.549  | 0.584    |              |
| Receivables to Total Assets           | 0.129    | 0.044      | 2.927   | 0.004**  |              |
| ROE                                   | -0.009   | 0.002      | -4.231  | 0.000*** |              |

Total Sum of Squares: 58553
R-Squared : 0.084566
F-statistic: 14.411 on 2 and 312 DF
theta: 0.4599

Residual Sum of Squares: 53602
Adj. R-Squared : 0.083761
p-value: 1.0323e-06
All variables are in logarithm.

Signif. codes: ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Source: Authors' own research.

Interesting were the results of model analysis for small-sized enterprises. It was found that other variables determine the relationship between the equity and assets in the group of companies not maintaining the golden rule of the financing and different in the group of companies that meet this rule. This enables the ability not only to identify the risks that lead to increase the level of risk of the insolvency, but also it allows to confront and eliminating its effects. Variables positively affecting the dependent variable include the lag Other Current...
Assets, Receivables to Assets, Profit/Loss before tax, and Ratio to Turnover to Assets. All of these variables can be considered as factors that obviously support the financial situation of the company (such as the pre-tax income / profit or loss before taxes), as well as influencing the enterprise positively within the working capital turnover (e.g. current assets). In addition the attention deserve the parameters that in a slightly longer term improve the financial situation of the company (and therefore enable the protection against the risk of insolvency), as the investment and additional funding of foreign capital.

Obviously, in difficult circumstances, these factors may impact negatively on the financial situation of the company, contributing to the deterioration of the relationship of equity funding assets (e.g. wrong investments in assets, or else the lack of resources to continue the already initiated investments). These conditions can therefore increase the level of risk of insolvency. In turn, the determinants affecting negatively the dependant variable include loans and other foreign financial resources. An interesting variable in this group of companies is negatively affected cash flow of the current year. This negative impact may be the result of a weaker financial situation already for small-sized enterprises, qualified to the group not maintaining the golden rule of financing. The variables affecting the behaviour of the golden rule of financing should be considered as risk factors which increase the risk of insolvency. It is therefore a subject to a distinctive control and financial discipline in order to eliminate the effects of the growth of this risk.

Quite a different situation is found in the case of Model 2, designed for small-sized enterprises, however where the golden rule of financing is maintained. Among the determinants of the E/FA relationship was found two: Receivables to Assets with a positive impact and ROE having a negative impact. The two variables relate to the previous period in relation to the dependent variable.

In the case of the first determinant with the simplified structure of assets the behaviour of the golden rule of the financing affects the participation of the receivables in the total assets. From the point of view of the part of assets that relatively more easily exchange for cash, it must be recognized that in solvent small-sized enterprises the relation of receivables and total assets were well managed and consequently the relationship of equity to assets was positively influenced.

Differently and at the first time surprisingly appeared the other determinant that is the negative impact of ROE on the degree of equity funding assets. However, if the company is treated as the "communicating vessels" (and otherwise it is impossible to analyse the problem of the risk of insolvency), it should be noted that such a variable character is expected. In case of dealing with loss than simply this relationship raises no doubts. Similarly, the situation would look like if the net profit growth will be slower than the gain in equity financing of fixed assets. From the point of view of the risk of insolvency, there can be low or negative profitability of equity treat as a less important (usually the first symptoms indicates-and rightly so-problems with maintaining liquidity) and it seems necessary to monitoring its variability in the context of eliminating the risks of insolvency. It must therefore be concluded that specific variables are for small businesses which do not meet the golden rule of the financing and for these entities that comply with this policy and therefore approve the first verified research hypothesis (HM). It can therefore be concluded that in the process of searching for the classification of companies at risk of insolvency and those which are not affected by insolvency-with due allowance for the risk of default- the separation of those companies using relationship E/FA seems reasonable. This is quite a rigorous approach to the assessment of the financial situation of the enterprises, it should be remembered, however, that small-sized enterprises are often carried out by a single entrepreneur, or jointly with a...
member of his family members and therefore the effects of this activity should make it worthy of funding the family entrepreneur. Often, in fact, it may happen that the entrepreneur secures business activity with private assets and, therefore, the identification of the factors that increase the risk of insolvency, as well as the confirmation that the company is not affected by insolvency are extremely important (from a scientific and functional point of view).

Table 2: The results of the model analysis for medium-sized enterprises.

Model 2a. The medium-sized enterprises don’t meet the golden rule of accounting (E/FA<1)

| Coefficients                      | Estimate | Std. Error | t value | Pr>|t| | Significance |
|-----------------------------------|----------|------------|---------|-----|-------------|
| (Intercept)                       | 0.246    | 0.220      | 1.119   | 0.264 |             |
| Debtors                           | -0.0001  | 0.000      | -1.715  | 0.087 |             |
| Current Assets to Total Assets    | -0.006   | 0.003      | -2.316  | 0.021 | *           |
| Receivables to Total Assets       | 0.010    | 0.004      | 2.710   | 0.007 | **          |
| Receivables to Assets (Lag: -1)   | -0.007   | 0.003      | -2.262  | 0.024 | *           |
| P/L after tax                     | -0.0002  | 0.000      | -1.772  | 0.077 |             |
| ROA                               | 0.008    | 0.003      | 2.494   | 0.013 | *           |
| Turnover Assets Ratio (Lag: -1)   | 0.001    | 0.000      | 3.494   | 0.001 | ***         |

Total Sum of Squares: 256.06 Residual Sum of Squares: 233.76
R-Squared : 0.0871 Adj. R-Squared : 0.086
F-statistic: 8.34 on 7 and 612 DF p-value: 9.43e-10
theta: 0.584 All variables are in logarithm

Model 2b. The medium-sized enterprises that meet the golden rule of accounting (E/FA>=1)

| Coefficients                      | Estimate | Std. Error | t value | Pr>|t| | Significance |
|-----------------------------------|----------|------------|---------|-----|-------------|
| (Intercept)                       | -1.921   | 2.541      | -0.756  | 0.452 |             |
| ROA (Lag: -1)                     | -0.103   | 0.040      | -2.578  | 0.012 | *           |
| ROE                               | -0.002   | 0.001      | -2.707  | 0.008 | **          |
| Shareholders’ Funds to Total Assets| 0.073    | 0.031      | 2.329   | 0.022 | *           |
| Turnover Assets Ratio (Lag: -1)   | 0.006    | 0.003      | 1.834   | 0.070 |             |

Total Sum of Squares: 3190.8 Residual Sum of Squares: 2634.6
R-Squared : 0.17432 Adj. R-Squared : 0.16394
F-statistic: 4.16969 on 4 and 79 DF, p-value: 0.0040822
theta: 0.3825 All variables are in logarithm

Signif. codes: ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Source: Authors’ own research.

At the same time, it should be pointed out that the developed models are properly conditioned from the formal point of view, and in the case of Model 1a. theta: 0.4118 that is the variability of individual effects of the type of random effects is responsible for 41.2% of the variation of the dependent variable. In the case of Model 1b. theta was 0.4599, and therefore the volatility of individual effects of the type of random effects is responsible for 46.0% of variation the dependent variable.

Similar calculations were carried out for medium-sized enterprises and the results are presented in Table 2. As of the model analysis for medium-sized enterprises which do not satisfy the condition of golden rule, it appears that the most strongly interacting variables, positively affecting the dependant variable should include lag of Ratio of Turnover to Assets,
Receivable to Total Assets, lag of ROA. The first two variables can be combined with the investment process and the structure of assets, in particular the participation of and change in the participation in this structure.

It can therefore be concluded that not without significance remains the level of assets (ongoing investment process, or reducing the value of assets, particularly in the current year), as well as the level of receivables, as well as the scale and pace of their impact (from the previous year, that is, following the availability of cash in the form of receivables in the current year). Differently, however, is the situation if one takes into account the determinant log_Receivables_To_Assets_Total, 0:1,1 of the current year.

This variable has already had a negative impact on the level of equity financing assets. This may indicate that the investment process - although in the previous year a favourable impact on the situation of the company- as a dynamically changing situation of an enterprise in a turbulent environment works negatively. The second variable with a negative effect on the dependent variable is ratio of Current Assets to Total Assets. This is another determinant of a structural nature. It can be considered that the improperly shaped relationship between Total Assets and Current Assets in the base year, results in the following year in the deterioration of the financial situation of the company and thus may contribute to increased risk of insolvency. In the group of medium-sized enterprises that meet the golden rule of accounting, consideration should be given to the three dependencies: the inverse relationship between the dependent variable and variable ROA, and ROE. It is an inherent relationship that indicates the decreasing possibility of cover equity assets in the event of loss which "competes" with the own funds of the company, instead of increasing the level of equity, the entity requires coverage. Therefore both ROA of the current year and the ROE of the base year are variables at which the particular attention should be paid, as it may indicate increased risk of insolvency of a group of companies in good financial condition. In turn, shortages of capital, can be effectively complemented by the Shareholders' Funds (the variable of base year ratio of Shareholders' Funds to Total Assets and at least in a short time, eliminate the increase in the risk of insolvency.

It was therefore concluded that the risk of insolvency in medium-sized enterprises which do not meet the golden rule of the financing may be identified using different variables than in medium-sized enterprises that meet the golden rule. It can therefore be concluded that the second test hypothesis has been verified positively (HS). Similarly, as in the case of models designed for small businesses, it should be pointed out that the developed models for medium-sized enterprises are properly conditioned from the formal point of view. In the case of Model 2a. theta was: 0.584, which is the variability of the individual effects of the type of the random effects responsible for approx. 58.4% of the variability in the dependent variable. In the case of Model 2b. theta was 0.383 and therefore the volatility of individual effects of the type of the random effects is responsible for approx. 38.2% dependent variation.

The rationale adopted for the research methodology - as an important variable in the identification of risk of insolvency the maintenance (or not) of the financial golden rule was established - was confirmed due to empirical research conducted among entrepreneurs. The model was developed based on the opinion of the entrepreneurs and the results are presented in Table 3.

Two factors are pointed out by entrepreneurs as being important in the identification of the risk of insolvency of enterprises where the first one is the bad debt management, meaning primarily excessively long periods of repayment that poorly match inflow of receivables. The second reported problem was the outflow of young, active workforce that is rather searching for a career opportunity in the larger municipalities or outside the country.
It can therefore be concluded that the issue of appropriate debt management, as well as the need for synchronization of receipts and expenditure, which should secure the current payments in a short run is crucial in identifying and reducing the risk of insolvency, in a short term (in relation to all groups of companies). However, in the longer term, consideration should be given to the different factors that can allow the identification of the risk of insolvency, which may further allow the entrepreneurs to make quick response, or spread out over time the operations (e.g. restructuring) and thus allow to avoid the insolvency.

### Table 3: The Model based on the respondents’ opinions.

**Model 3. The factors that increase the risk of insolvency in the opinions of entrepreneurs.**

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>z value</th>
<th>p - value</th>
<th>Odds Ratio</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-1.810</td>
<td>0.416</td>
<td>-4.356</td>
<td>0.000</td>
<td>0.164</td>
<td>***</td>
</tr>
<tr>
<td>Bad debt management</td>
<td>1.164</td>
<td>0.628</td>
<td>1.854</td>
<td>0.064</td>
<td>3.201</td>
<td>.</td>
</tr>
<tr>
<td>Outflow of work force</td>
<td>-1.451</td>
<td>0.728</td>
<td>-1.993</td>
<td>0.046</td>
<td>0.234</td>
<td>*</td>
</tr>
</tbody>
</table>

Pseudo $R^2$: 0.080
Chi$^2$: 6.745, $p = 0.039$

Signif. codes: ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Source: Authors' own research.

In addition, the presented results of the study are a follow-up to a wider research project, which concluded that "in SMEs diversified from the point of view of the level of the creditors, various factors affecting the increase in the risk of insolvency can be highlighted. It was found that keeping creditors below the median enables the company to improve its financial situation" (Franc-Dąbrowska, Porada-Rochoń, Suwala, 2016). Those authors also found, that "the research has enabled the identification of determinants which makes it possible to define the positive influence on the financial condition of enterprises, measured by the level of cash flow according to for medium-sized enterprises" (Franc-Dąbrowska, Porada-Rochoń, Suwala 2015).

### References


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